Impact of size segregation on the sediment mobility during bedload transport.

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Introduction - Context

During bedload transport:
- Necessity to estimate sediment flux
- Poorly sorted sediment → size sorting
- Impact on the sediment flux. Which one? Are we able to characterise them?

Pictures of armouring on the field resulting from size-segregation
Numerical Model

Discrete Element Method (DEM) coupled with a 1D turbulent fluid model (Maurin et al. 2015, 2016):
- DEM (code YADE): Lagrangian method based on contact between particles
- Fluid: 1D vertical turbulent fluid flow based on mixing length closure

Granular phase, for each particle $p$:

$$m_p \frac{d^2 \vec{x}_p}{dt^2} = \vec{f}_c^p + \vec{f}_g^p + \vec{f}_f^p$$

$$\mathcal{I}_p \frac{d\vec{\omega}_p}{dt} = \vec{\tau} = \vec{x}_c \times \vec{f}_c^p$$

Fluid phase:

$$\rho^f \epsilon \frac{\partial \langle u_x \rangle^f}{\partial t} = \frac{\partial S^f_{xz}}{\partial z} - \frac{\partial R^f_{xz}}{\partial z} + \rho^f \epsilon g_x - n \langle f_D \rangle$$

- $\vec{f}_f^p$: fluid forces over particles
- $\vec{f}_b^p$: Buoyancy force
- $\vec{f}_D^p$: Drag force

$$\vec{f}_D^p = \frac{1}{2} \rho^f \frac{\pi d^2}{4} C_D \| \vec{u}^f - \vec{v}^p \| \left( \vec{u}^f - \vec{v}^p \right)$$
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\]

Reynolds shear stress:

\[
R_{xz}^f = \rho_f \nu_t \frac{d \langle u_x \rangle^f}{dz}
\]

\( \nu_t \): turbulent viscosity computed with a mixing length
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**Introduction**

**Numerical Model**

- **Transport law**
- **Effect**

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**Numerical setup**

Geometrical parameters:
- 3D bi-periodic domain
- Slope: 10%
- $d_l/d_s = 2$ (6 mm / 3 mm)
- $H = 8d_l$
- Shields Number: $\theta = \frac{\tau_f}{(\rho_p - \rho_f)gd_l}$

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Simulations

$\nu^p_x$: exponential decrease with depth

Total concentration: $\phi = \phi_s + \phi_l$

Transport rate:

$$Q_s = \int \phi \nu^p_x dz$$

$$Q^*_s = \frac{Q_s}{\sqrt{\left(\frac{\rho_p}{\rho_f} - 1\right) g \cos(\alpha) \bar{d}^2}}$$

$\bar{d}$: mean surface diameter
Transport. Comparison Monodisperse and \( N_l = 2 \)

Comparison Mono, \( N_l = 2 \):
- \( Q_{s}^{\text{mono}} = 15.77\Theta^{1.88} \)
- \( Q_{s}^{N2} = 21.59\Theta^{1.88} \)
- Transport 37\% more efficient
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Local transport: $q_s^i = \phi^i v_x^i$
- Both small and large particles are transported
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- Big shield: big effect
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- Not a rugosity effect
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- Necessity to transport small particles
- Not a rugosity effect
Small conclusion

Summary:
- Model: reproduce an increase of mobility in bidisperse case
- Small particles need to be transported
- Small and large particles take part of the increase

Why are small particles sometimes transported?
- Fluid effect: fluid shear stress sufficient to transport small particles?
- Granular effect: how, what effect?
Fluid effect?

Dimensionless fluid shear stress: \( \tau^f* = \frac{\tau^f}{\sqrt{(\rho^p/\rho^f - 1)g\cos(\alpha)d^3}} \), \( d \) local diameter

\( \tau^f* \) too small to be responsible for the mobility of small particles.
Not a fluid effect \( \Rightarrow \) granular effect
\( \mu(I) \) rheology makes a link between:

- the stress state of a granular material \( (\mu = \frac{\tau^P}{P^P}) \)
- the dynamical state of the granular media \( (I) \)

Inertial number:

\[
I = \frac{d\gamma}{\sqrt{P^p/\rho^p}} = \frac{t_{\text{micro}}}{t_{\text{macro}}}
\]

GDR midi (2004):

- \( \mu < \mu_s \) : no motion (for glass \( \mu_s = 0.38 \))
- \( \mu \geq \mu_s : \mu = \mu(I) \) (bijection)
Granular stress state

$\Theta = 0.3$, comparison monodisperse and $N_l = 2$ configuration.

Same stress state in both configurations
Granular stress state

\( \Theta = 0.3 \), comparison monodisperse and \( N_l = 2 \) configuration.

- Same stress state in both configurations
- \( \mu = \frac{\tau_p}{P_p} \), same friction coefficient
- \( \mu(I) \) rheology : same \( I \)
- \( I_{\text{mono}} = I_{N2} \)

\[ \Rightarrow \frac{d_l \dot{\gamma}_{\text{mono}}}{\sqrt{P_p / \rho_p}} = \frac{d \dot{\gamma}_{N2}}{\sqrt{P_p / \rho_p}} \]

\[ \Rightarrow \dot{\gamma}_{N2} = \frac{d_l}{d} \dot{\gamma}_{\text{mono}} \]
Application of $\mu(I)$ rheology

- First zone ($0 < z < z_1$): $\mu < \mu_s$ no motion
**Application of $\mu(I)$ rheology**

- **First zone** ($0 < z < z_1$): $\mu < \mu_s$ no motion
- **Second zone** ($z_1 < z < z_2$): $\dot{\gamma}_{N2} = \frac{d_l}{d_s} \dot{\gamma}_{mono} = 2\dot{\gamma}_{mono}$
  
  $\Rightarrow v_{N2}(z) - v_{N2}(z_1) = 2(v_{mono}(z) - v_{mono}(z_1))$
  
  $v_{N2} = 2v_{mono}$

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Application of $\mu(I)$ rheology

- First zone $(0 < z < z_1)$: $\mu < \mu_s$ no motion
- Second zone $(z_1 < z < z_2)$: $\dot{\gamma}_N^2 = \frac{d}{d_s} \dot{\gamma}_{mono} = 2\dot{\gamma}_{mono}$
  \[ \Rightarrow v_N^2(z) - v_N^2(z_1) = 2(v_{mono}(z) - v_{mono}(z_1)) \]
  \[ v_N^2 = 2v_{mono} \]
- Third zone $(z_2 < z)$: $\dot{\gamma}_N^2 = \dot{\gamma}_{mono}$ \[ \Rightarrow v_N^2(z) - v_N^2(z_2) = v_{mono}(z) - v_{mono}(z_2) \]
  \[ v_N^2 = v_{mono} + \Delta v(z_2) \]
Conclusions

- Increased mobility: granular effect
- Can be explained by the $\mu(I)$ rheology
- Small particle are more mobile due to rheological effect
- Boundary problem for the large particles at the top
- If interface large/small is below the limit $\mu = \mu_s$: no effect
Making an idealized 3 layers model: small no moving, small in motion, large in motion.

- Compute the width of the small particles layer in motion.
- Estimate the slip velocity.
- Estimate the enhanced sediment transport.
Perspective 2

Is there a modification of the rheology due to the small particles? Same $\mu \Rightarrow$ Same $I$?