

# Modeling particle's shape in granular materials with a Level Set-Discrete Element Method

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# Granular materials @ Irstea

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## Rockfill for rockfill dams



*The Rock Manual, CIRIA*

Serre-Ponçon rockfill dam, 123 m

<https://www.edf.fr/edf/accueil-magazine/serre-poncon-cumule-les-superlatifs>

# Granular materials @ Irstea

## Soils for earthfill dams



[www.elyskiphire.co.uk](http://www.elyskiphire.co.uk)

Moreau dam under construction, half-2016

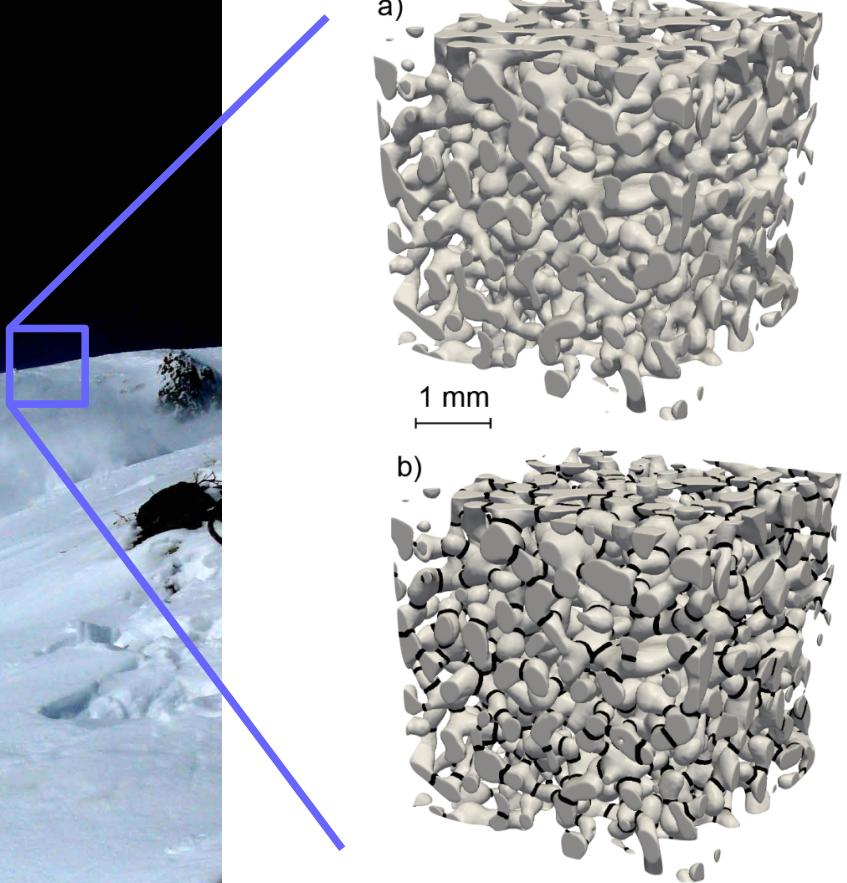
Photo G<sup>2</sup>DR/Irstea

# Granular materials @ Irstea

## Snow for snow avalanches



Hervé Bellot/Irstea



Hagenmuller et al.,  
*The Cryosphere*, 2015

# Granular materials @ Irstea

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## Bedload/Sediments for solid transport



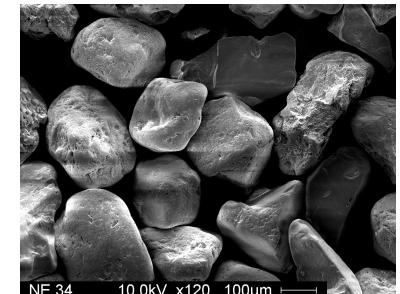
Maurice Meunier/Irstea



Mickael Lagouy/Irstea

# Hence the Discrete Element Method (DEM)

Numerical simulation of a granular material



[www.elyskiphire.co.uk](http://www.elyskiphire.co.uk)

Laboratoire Navier

State of the set  
of particles  
in interaction,  
at  $t = i \times \Delta t$

Contact treatment:  
detection,  
relative displacements

Contact forces from  
relative displacements

State of the set of particles at  $t = (i+1) \times \Delta t$

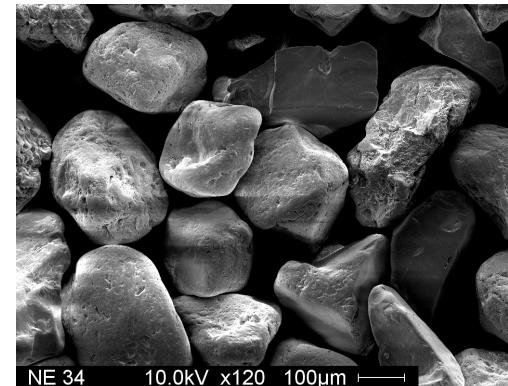
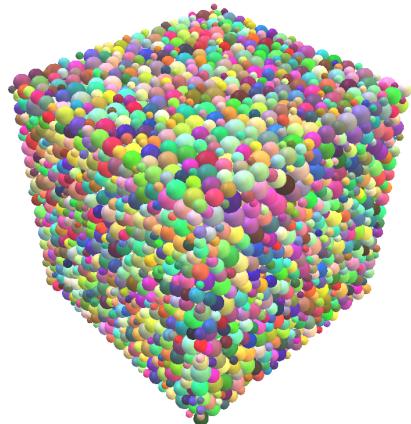
Particles movements:  
rigid bodies equations,  
during the time  $\Delta t$

2<sup>nd</sup> Newton's law  
and Euler's equations

# Discrete Elements

## Classical shapes for Discrete Elements

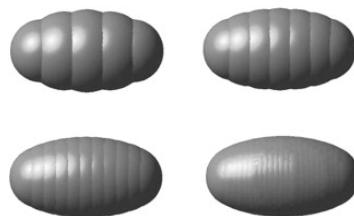
- Spheres



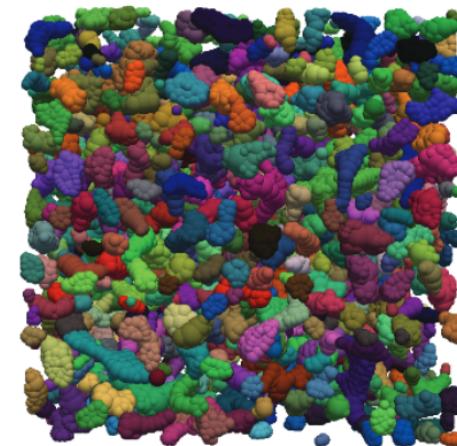
Sand

Laboratoire Navier

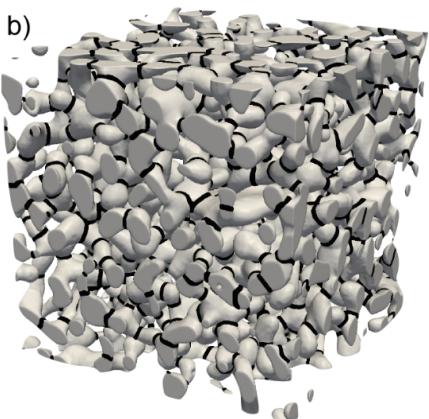
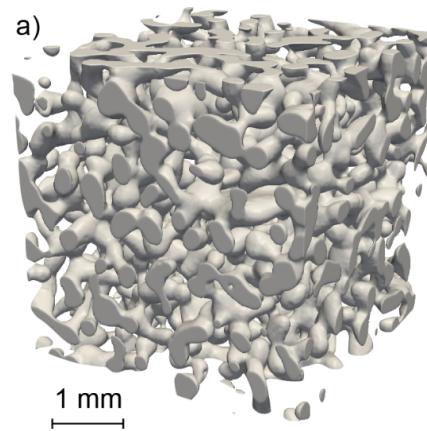
- Rigid aggregates (Clumps) of spheres



Höhner et al.,  
*Powder Technology*, 2011



Mede et al., *Powders & Grains* 2017



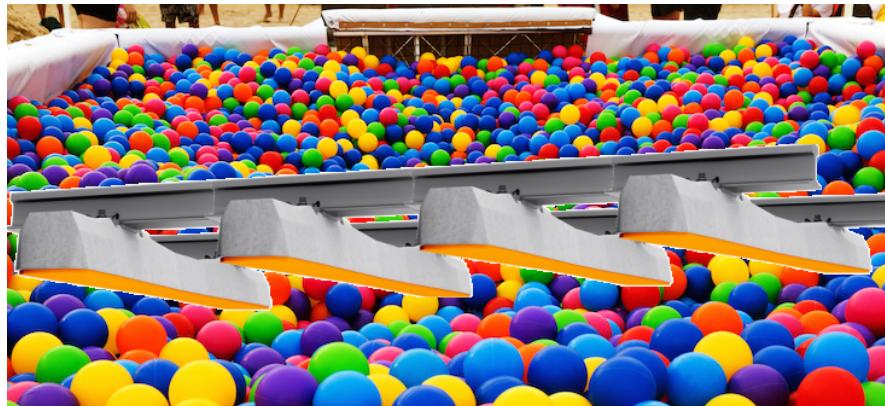
Snow

Hagenmuller et al., *The Cryosphere*, 2015

# Role of shape

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Wanna go for a 300 km/h TGV ride ?



OR  
?

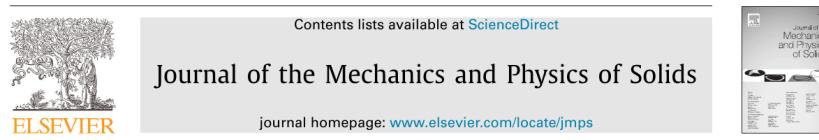


[www.getzner.com](http://www.getzner.com) ; [londonist.com](http://londonist.com)  
(adapted)

Public Domain, <https://commons.wikimedia.org/>

# A new shape descriptor: LS-DEM

## Level Set – Discrete Element Method (LS-DEM) for a better shape description ?



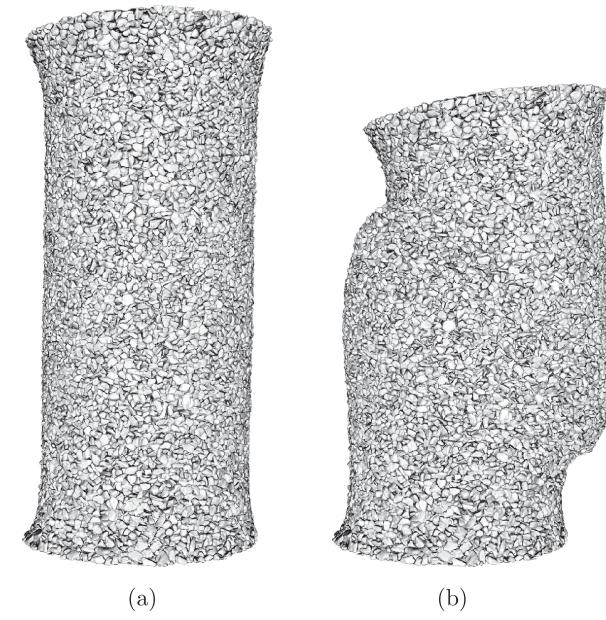
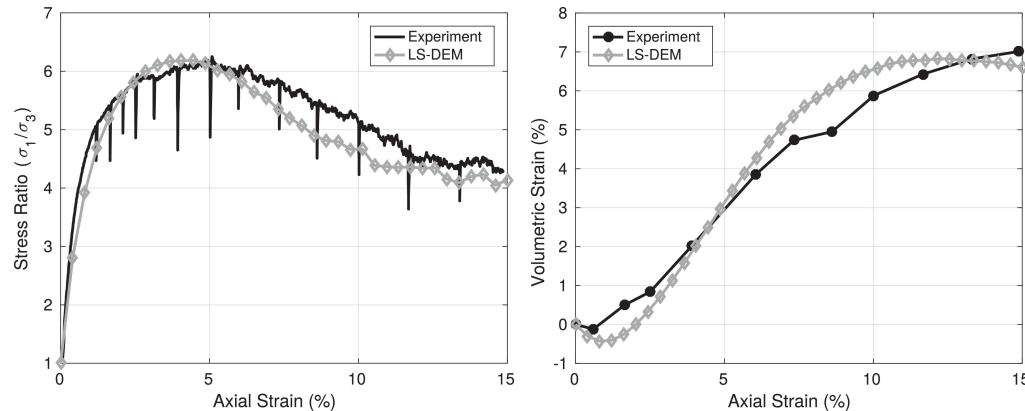
All you need is shape: Predicting shear banding in sand with LS-DEM



Reid Kawamoto<sup>a</sup>, Edward Andò<sup>b</sup>, Gioacchino Viggiani<sup>b</sup>, José E. Andrade<sup>a,\*</sup>

<sup>a</sup>Division of Engineering & Applied Science, California Institute of Technology, Pasadena, CA 91125, USA

<sup>b</sup>Grenoble-INP / UJF-Grenoble 1 / CNRS UMR 5521, Laboratoire 3SR, Grenoble, France

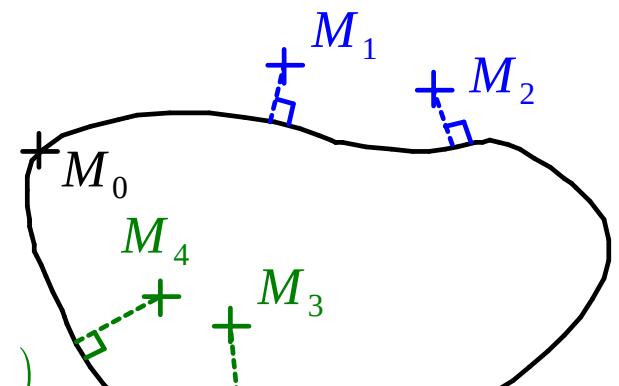


$$\phi_S(M_1) = \phi_S(M_2) > 0$$

$$M_1 \\ M_2$$

$$\phi_S(M_0) = 0$$

$$\phi_S(M_4) \\ = \phi_S(M_3) < 0$$

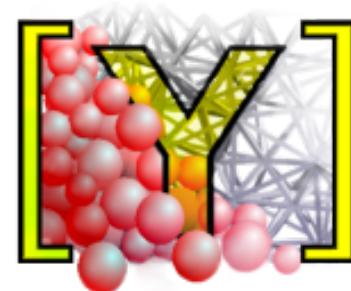


- Shape description from *signed distance function*  $\phi_S(\vec{x})$

# Outline

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- Level Set – Discrete Element Method and its (ongoing) implementation in YADE
- Examples and Computational aspects



[www.yade-  
dem.org](http://www.yade-dem.org)

# YADE implementation overview

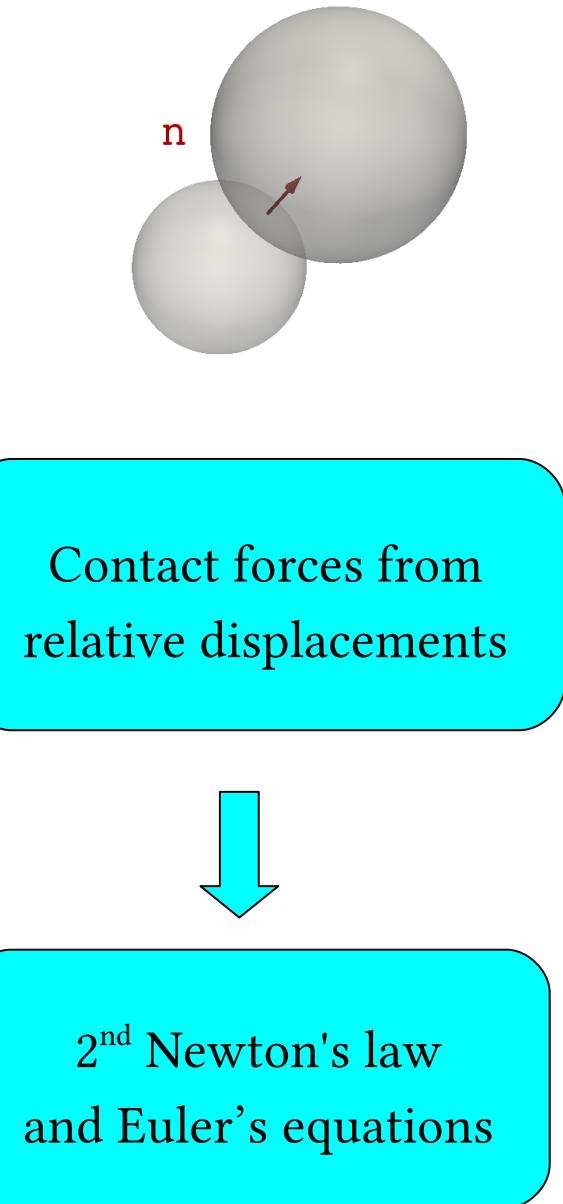
## New code and old code

class LevelSet  
( $\leftarrow$  Shape)

class Bo1\_LevelSet  
     $\Rightarrow$  Axis-aligned bounding box  
class Ig2\_LevelSet\_LevelSet\_ScGeom  
     $\Rightarrow \vec{n}; u_n; \Delta u_t$

State of the set  
of particles  
at  $t = i \times \Delta t$

Contact treatment:  
detection,  
relative displacements

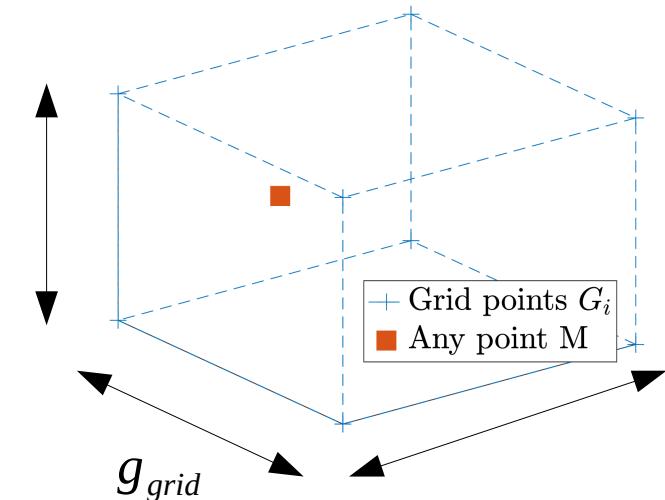
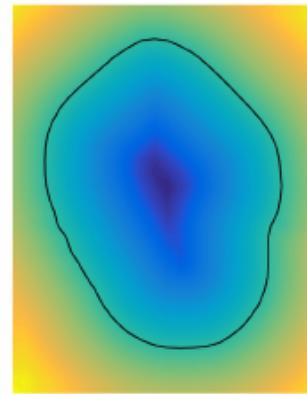
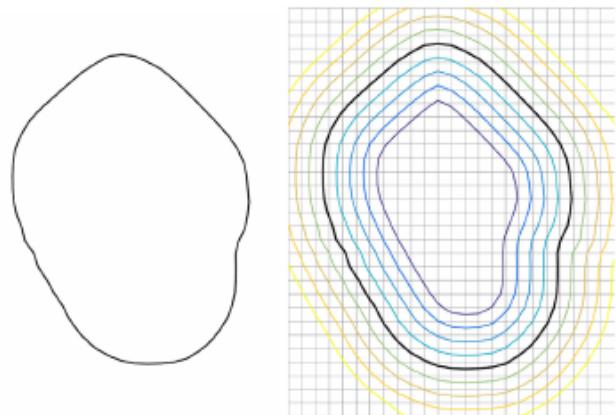


State of the set of particles at  $t = (i+1) \times \Delta t$

# The Level Set – Discrete Element Method

## Discrete description of signed distance function $\phi_s$

Storing  $\phi_s$  values on a particle-centered grid

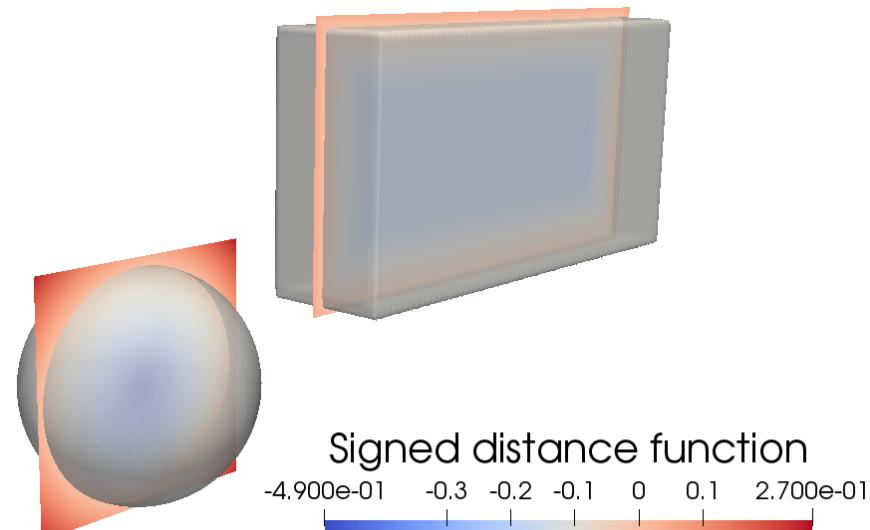


Kawamoto et al., *J. of the Mechanics and Physics of Solids*, 2016

## Discrete (Voxellised) description of particles volumes

$$\bullet \quad V_{grain} = \int_{V_{grain}} dV \approx \sum_{\text{'inside' voxels}} g_{grid}^3$$

- and so on for all inertial quantities



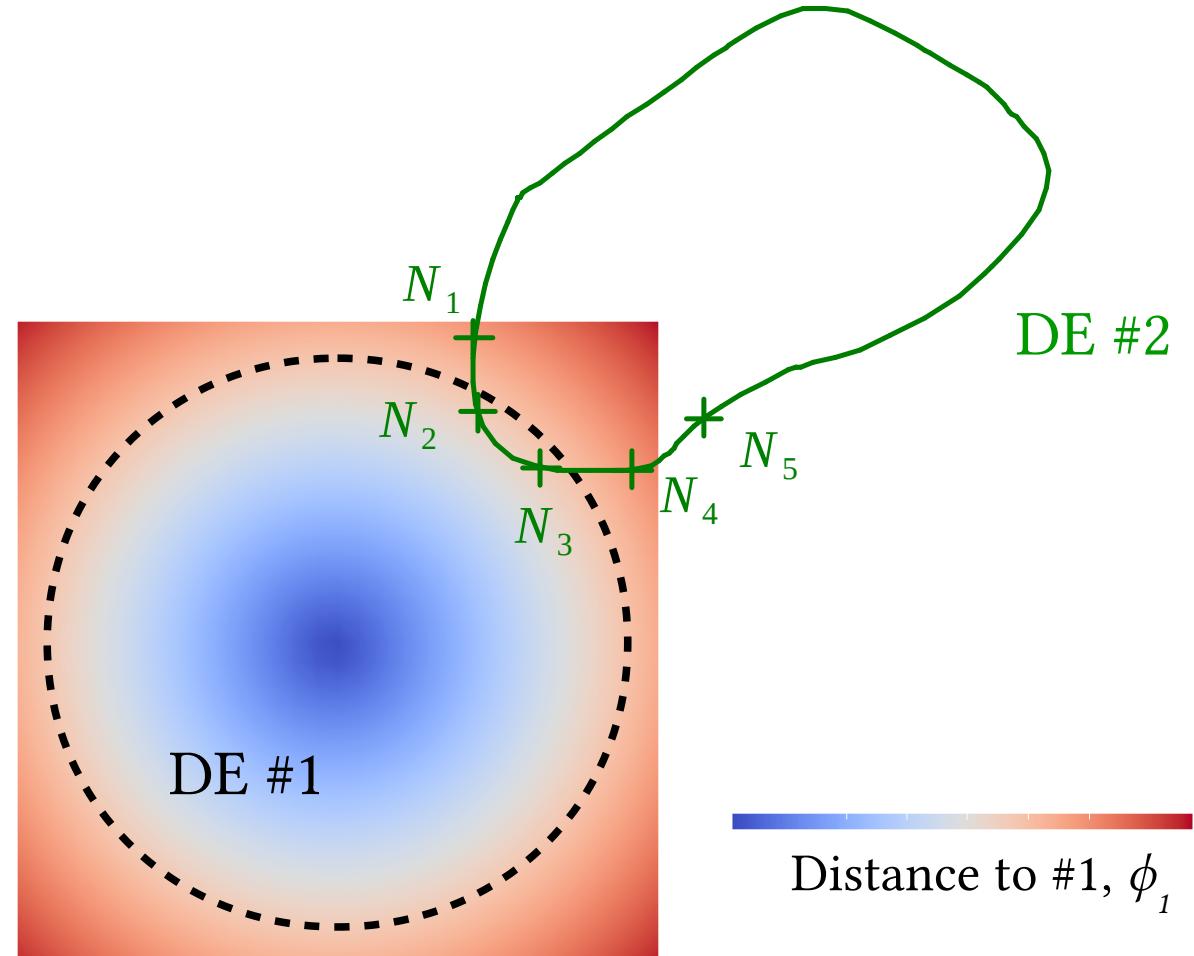
# The Level Set – Discrete Element Method

Contact treatment: “master-slave” with boundary nodes

- $\phi_1$  field
- boundary nodes  $N_i$  of 2, along  $S_2$

$$\left\{ \begin{array}{l} \min_{N_i} \phi_1(N_i) < 0 ? \\ \vec{n} = \nabla \phi_1 \end{array} \right.$$

accounting for rigid  
bodies' transformations



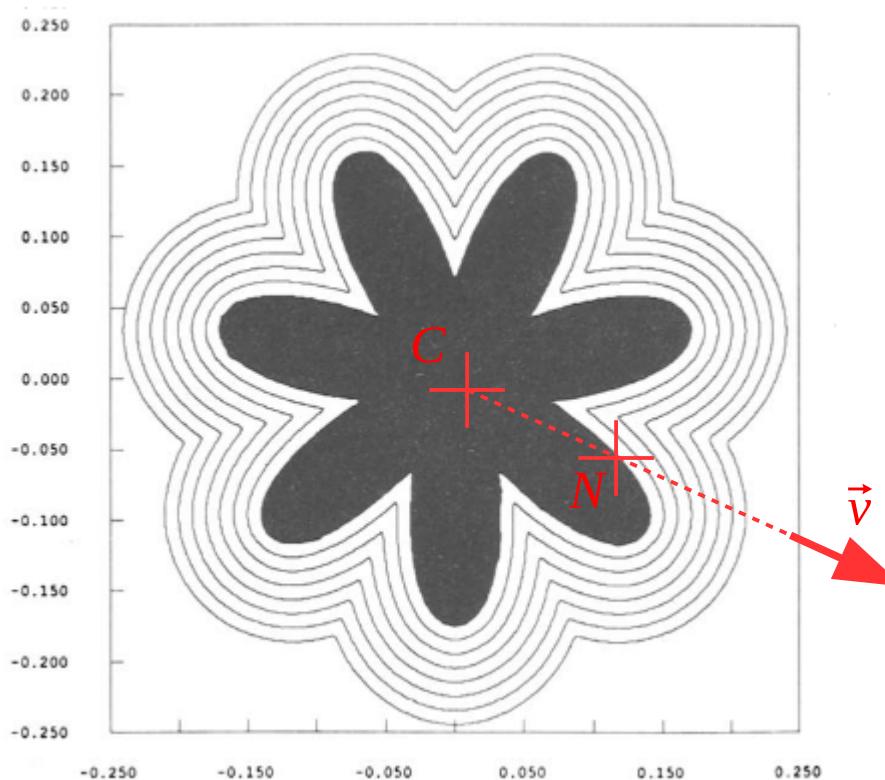
Objective ??

# The Level Set – Discrete Element Method

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Ray tracing of boundary nodes  $N_i$

Once, at DE creation:



Osher & Sethian, *J. of Computational Physics*, 1988

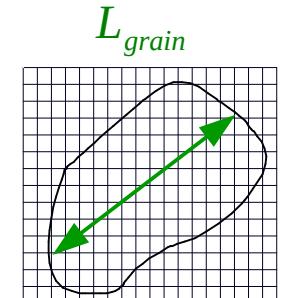
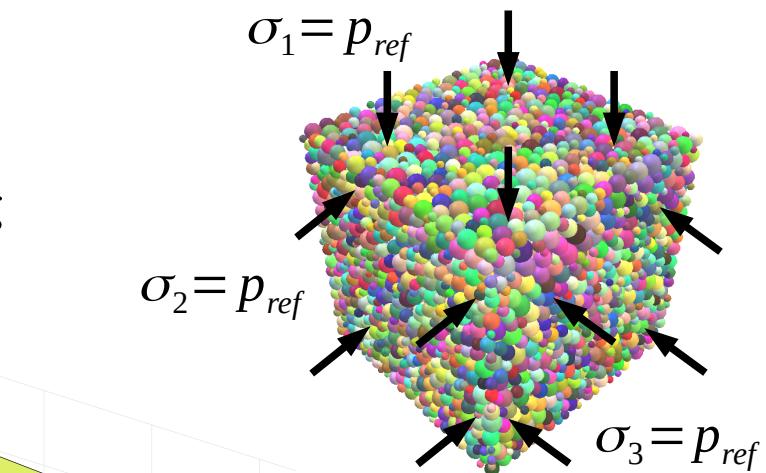
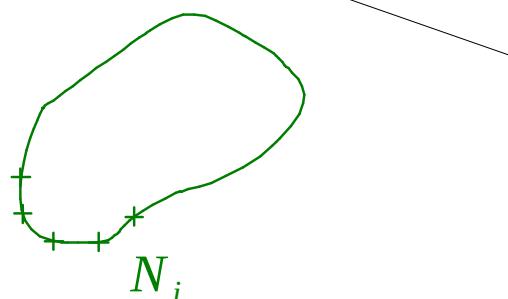
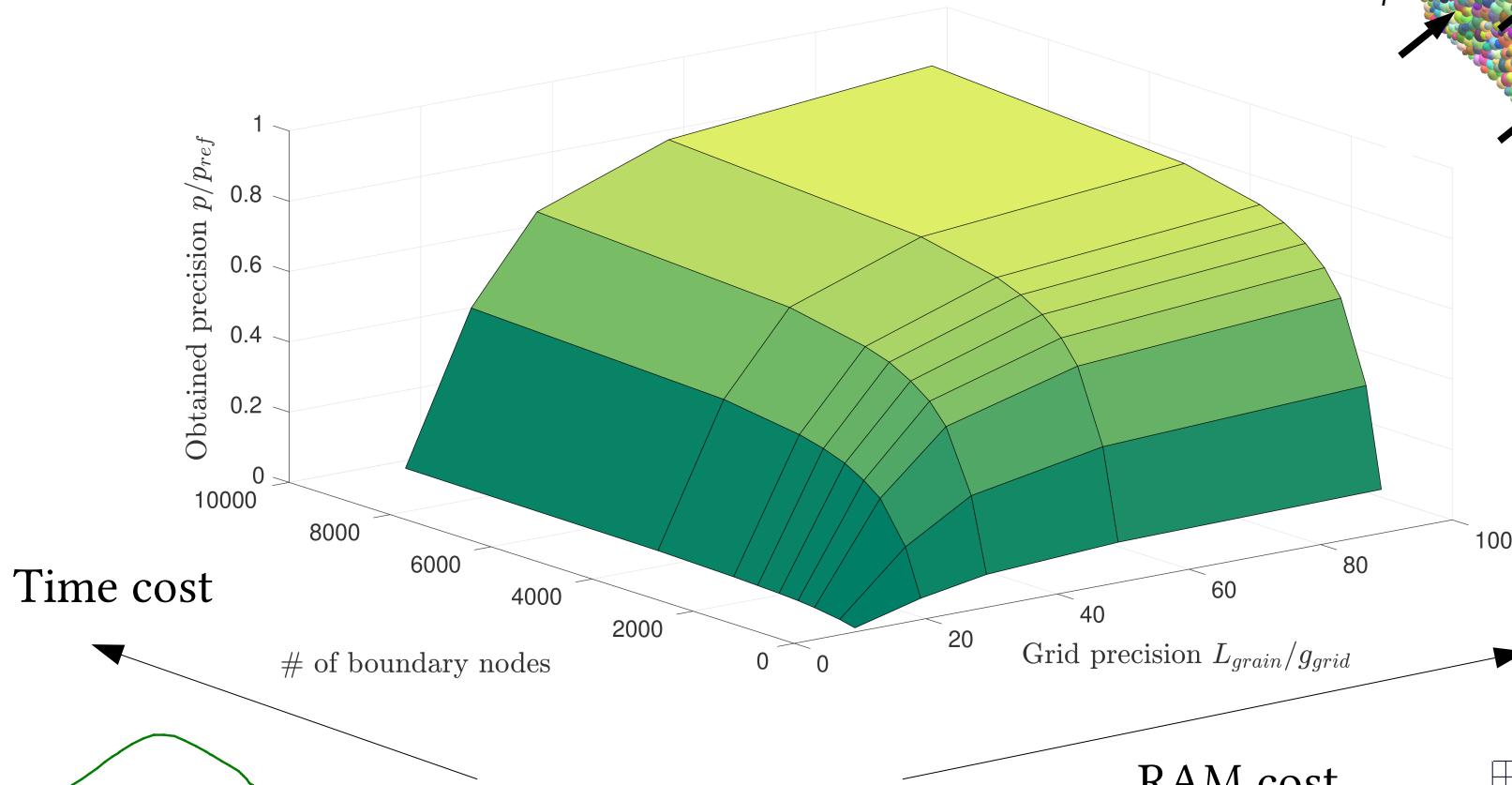
For  $N = C + k \vec{v}$ ,

$$\phi_S(N) = 0 \Leftrightarrow a_3 k^3 + a_2 k^2 + a_1 k + a_0 = 0$$

# The Level Set – Discrete Element Method

## Precision of Level Set – DEM

Isotropic configuration of 8000 spheres in a dense packing

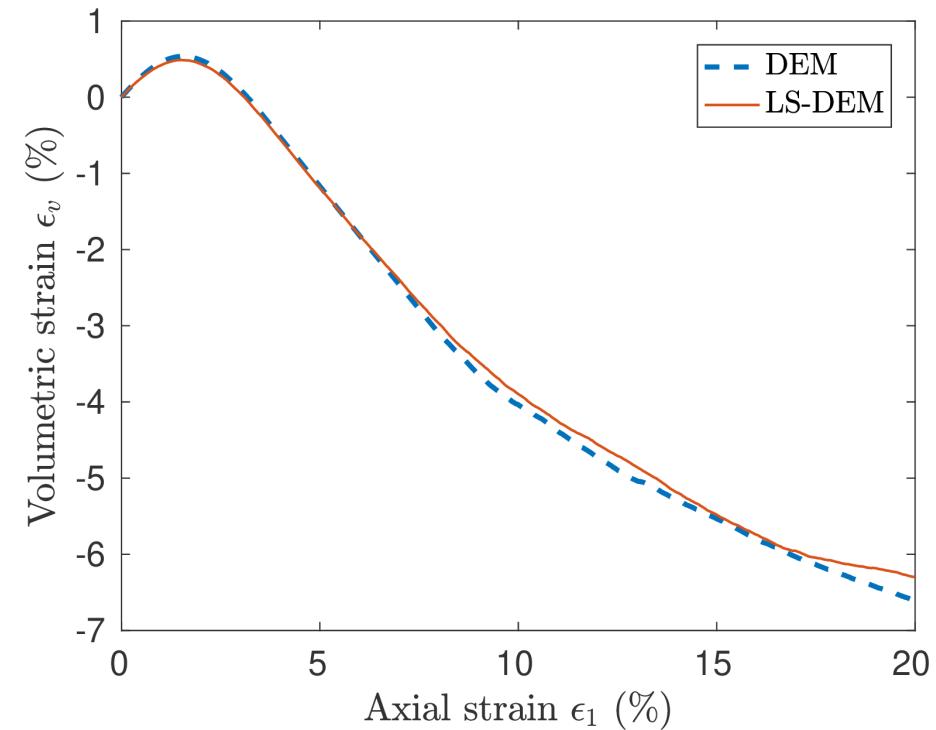
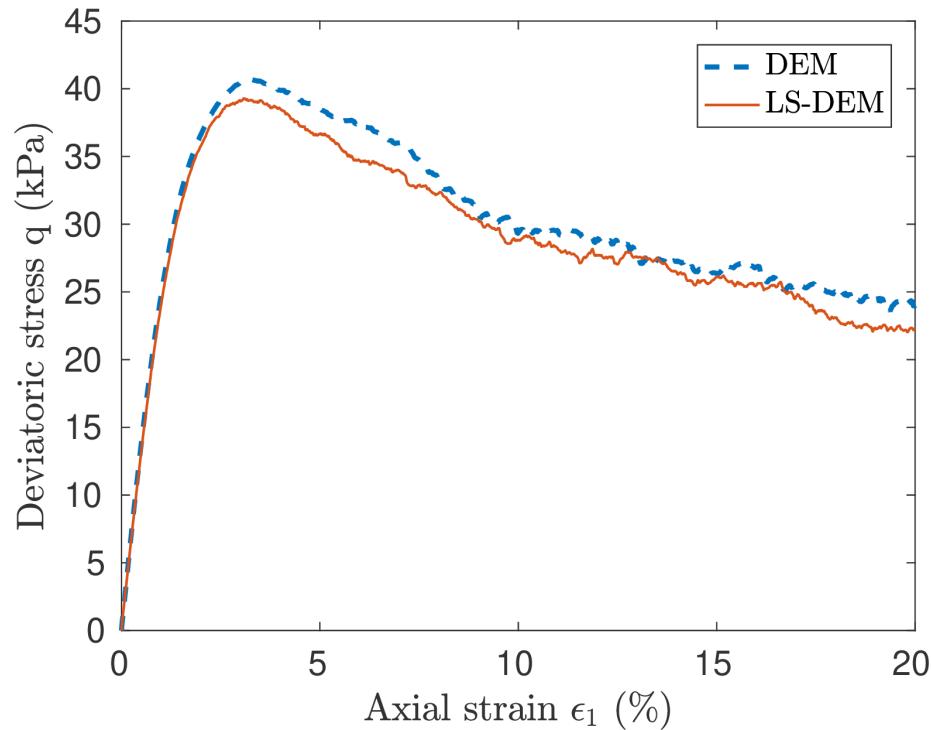
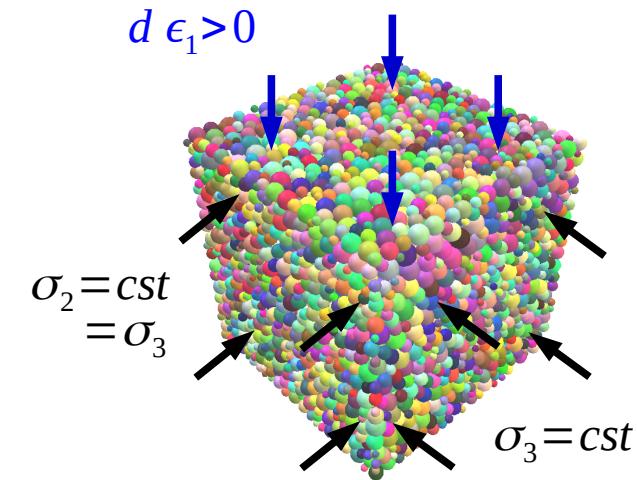


# Validation example: LS-DEM vs DEM

## Triaxial test

Dense packing of 8000 spherical DE

- $50^2=2500$  boundary nodes
- $L_{grain}/g_{grid}=90$

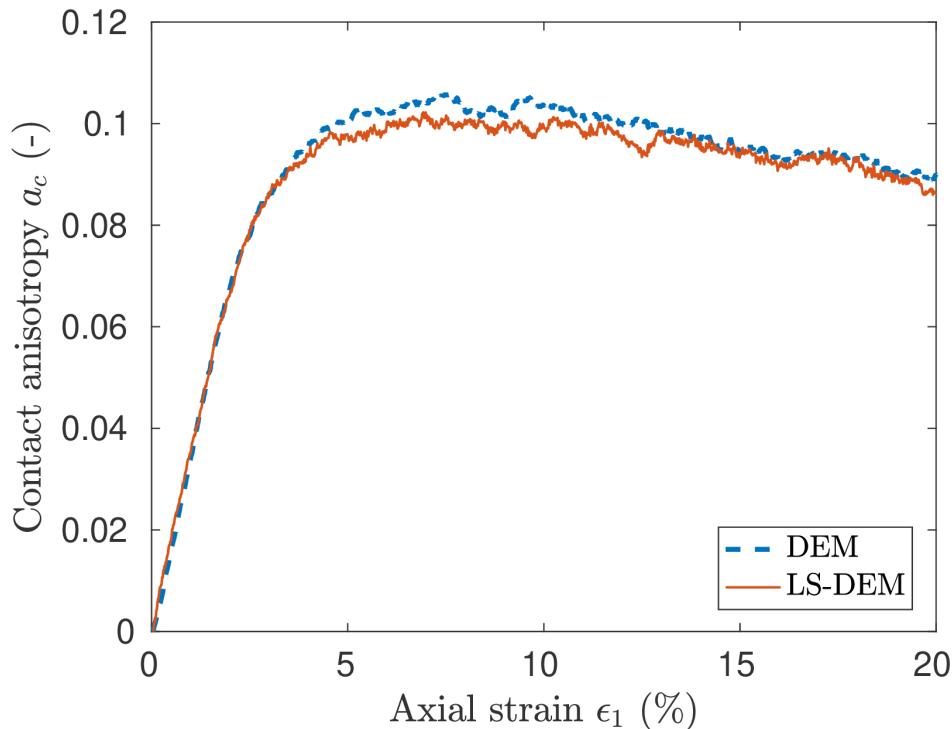


# Validation example: LS-DEM vs DEM

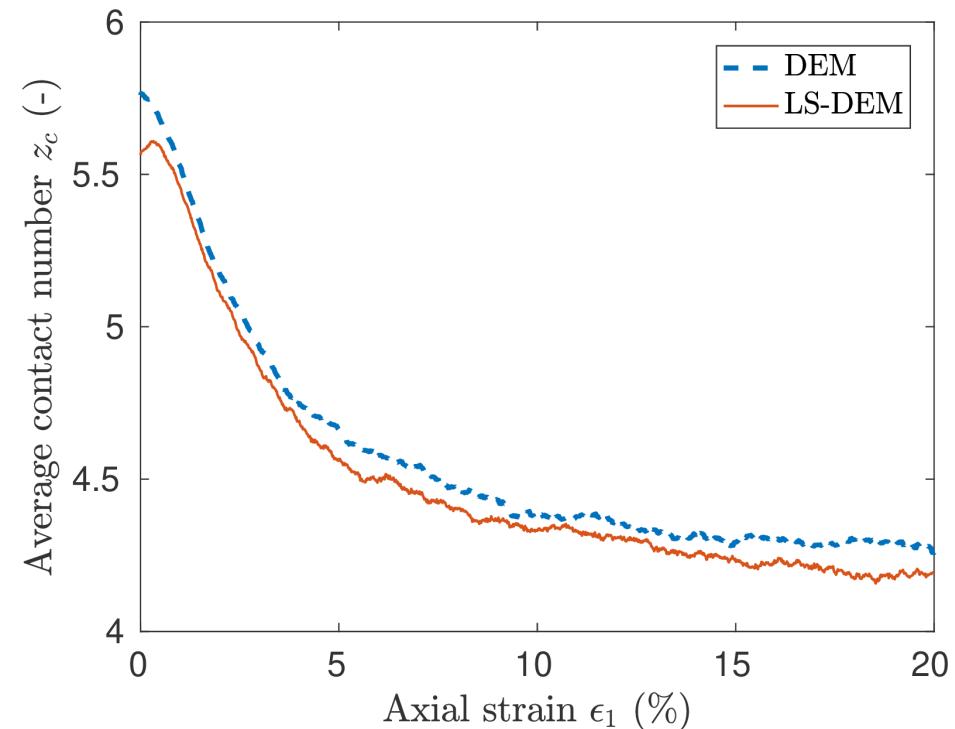
## Triaxial test

Dense packing of 8000 spherical DE

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- $L_{grain}/g_{grid}=90$



$$a_c = \text{dev} \left( \frac{1}{N_c} \sum_c \vec{n} \otimes \vec{n} \right)$$

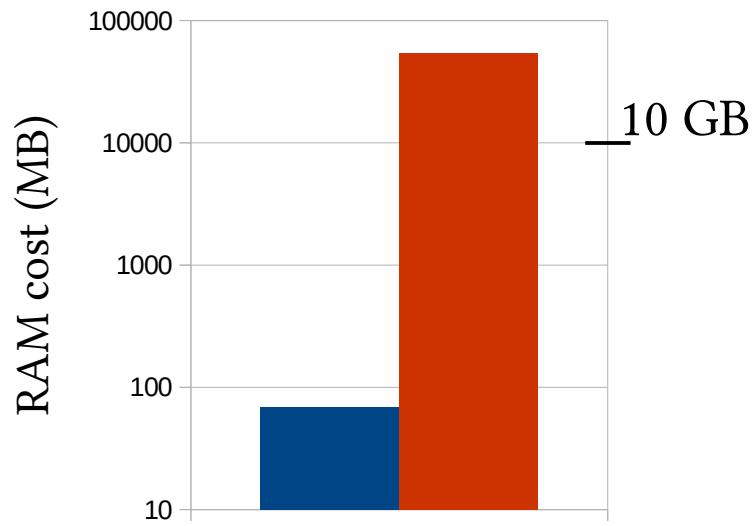


# Validation example and computational cost

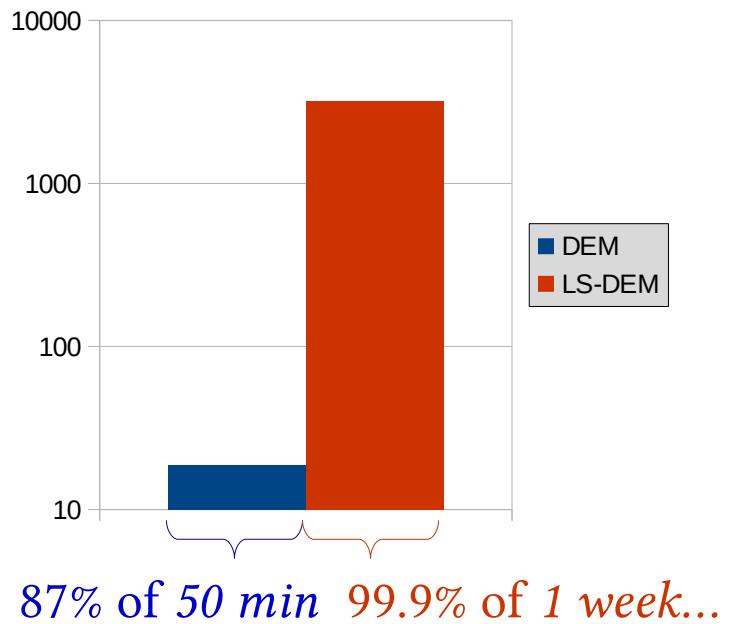
## Triaxial test

Dense packing of 8000 spherical DE

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Time cost for contact treatment (ms/it.)



Optimal choices ???

# Some calissons ?

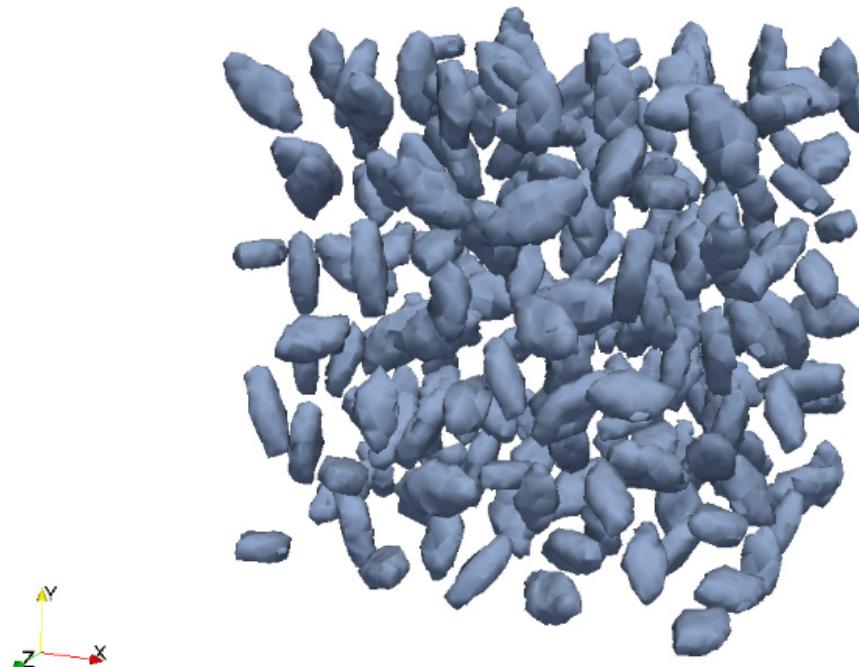
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## Superellipsoids (preliminaries)

$$f = \left( \left| \frac{x}{r_x} \right|^{\frac{2}{\epsilon_1}} + \left| \frac{y}{r_y} \right|^{\frac{2}{\epsilon_1}} \right)^{\frac{\epsilon_1}{\epsilon_2}} + \left| \frac{z}{r_z} \right|^{\frac{2}{\epsilon_2}} - 1$$



[www.calisson.com](http://www.calisson.com)

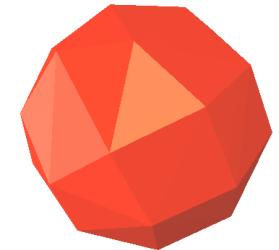
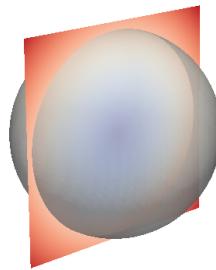


particleStress



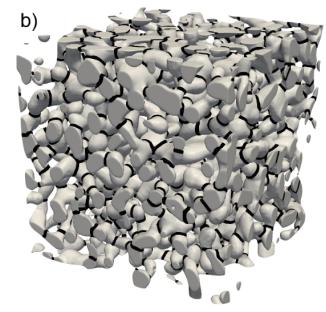
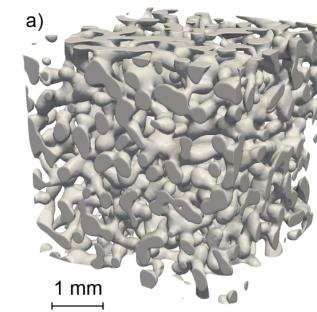
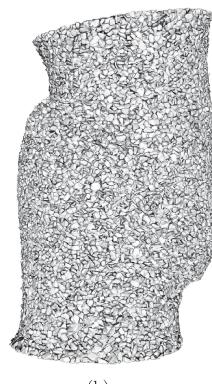
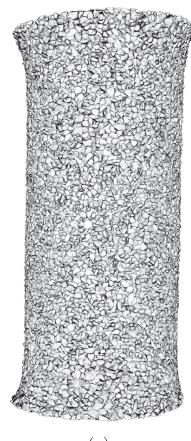
# Conclusions & Perspectives

- LS-DEM and new shapes coming soon in YADE ?
- suited for any shape (no convexity required)



Level Set-DEM vs Convex polyedron

- apply to real shapes from
  - laser scanning on rockfill materials (2020)
  - CT scans on other materials ?



# Merci !

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- à la Région Sud, projet LS-ENROC
- Stéphane Bonelli (Irstea, RECOVER),  
Cédric Galusinski et Frédéric Golay (Université de Toulon, IMATH)
- et vous pour votre attention !



# Merci !

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Les Inconnus, *Youpi Matin*

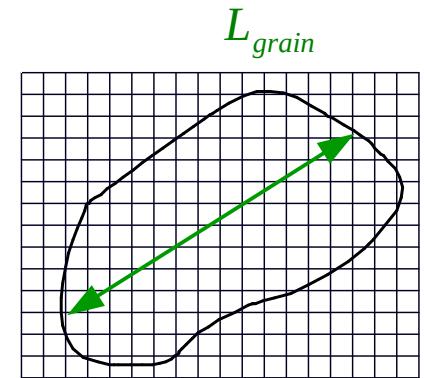
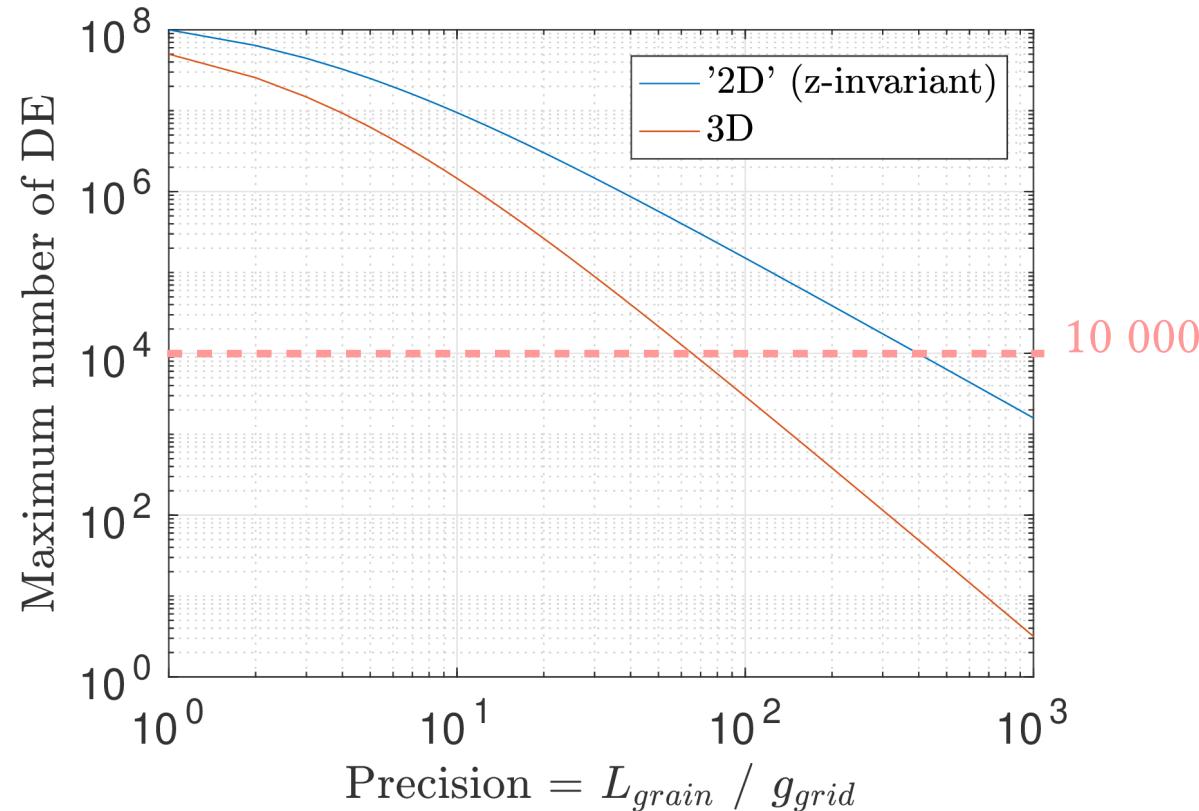
# The Level Set – Discrete Element Method

## Memory cost

For storing (with C<sup>++</sup> ↔ Python exposure)

```
std::vector<std::vector<std::vector<double>>> distanceValues
```

- machine with 64 GB of RAM
- an *average* memory cost of 20 bytes / distance value

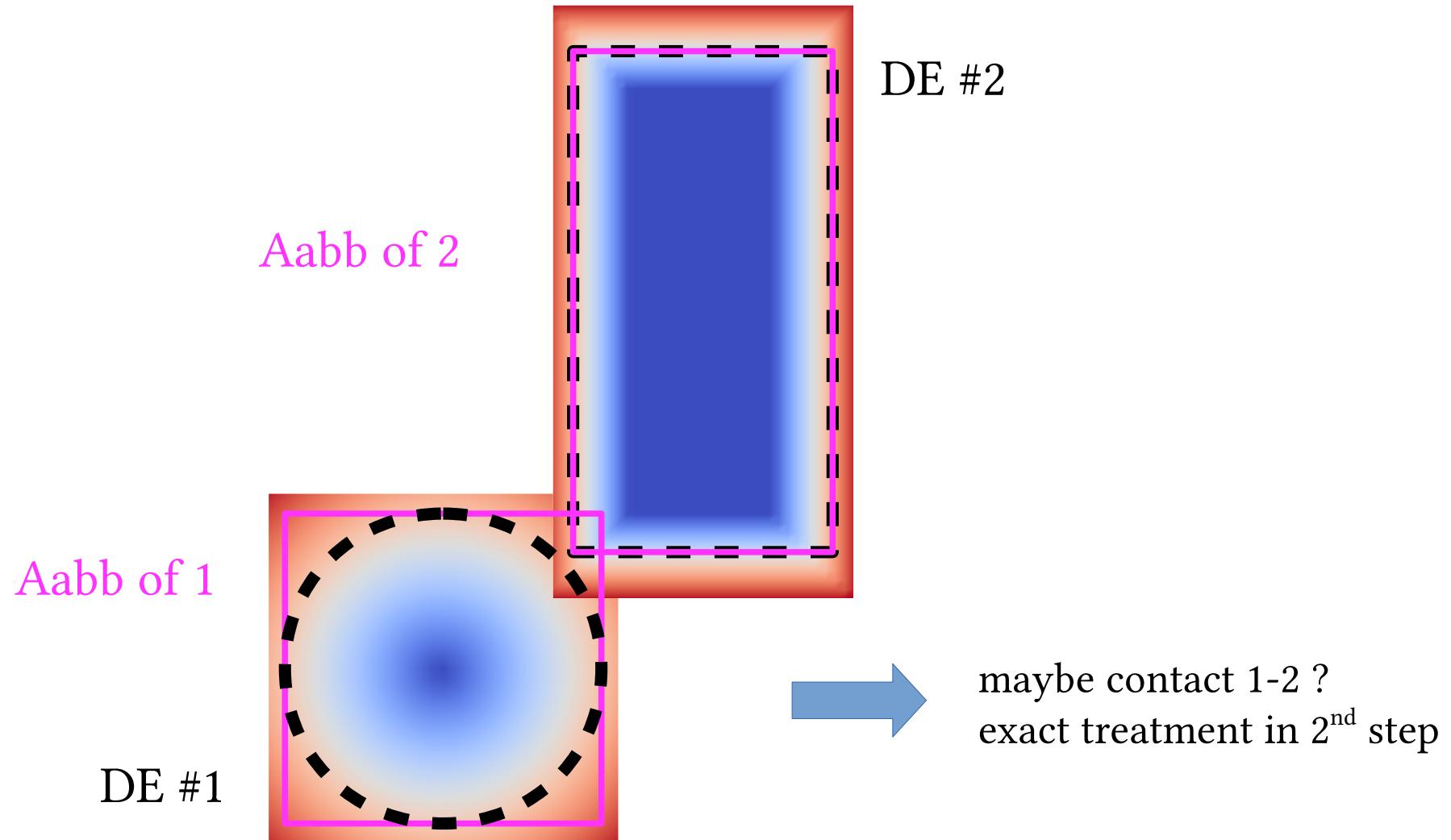


# The Level Set – Discrete Element Method

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Contact treatment: 1<sup>st</sup> step (approximate)

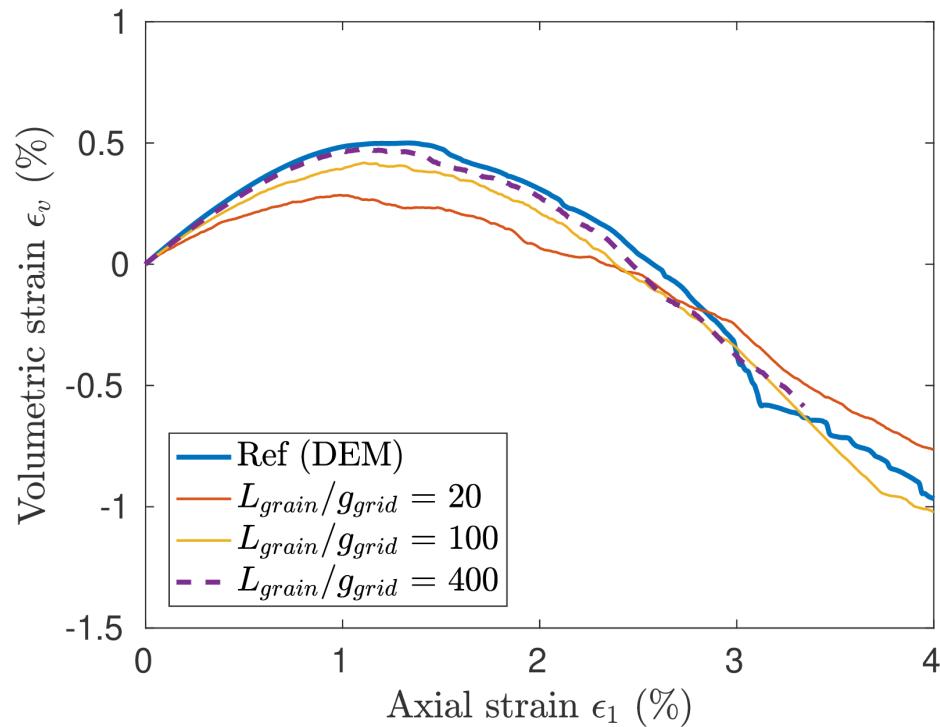
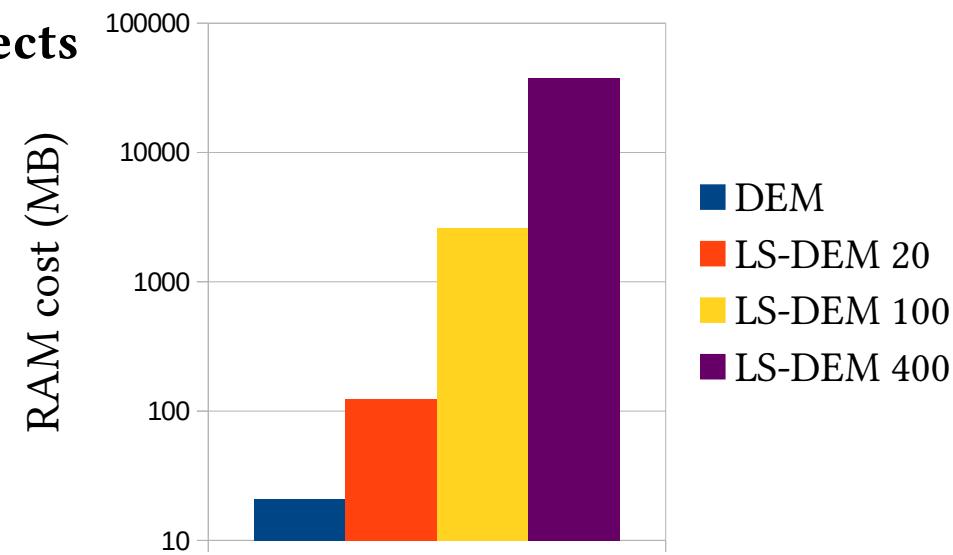
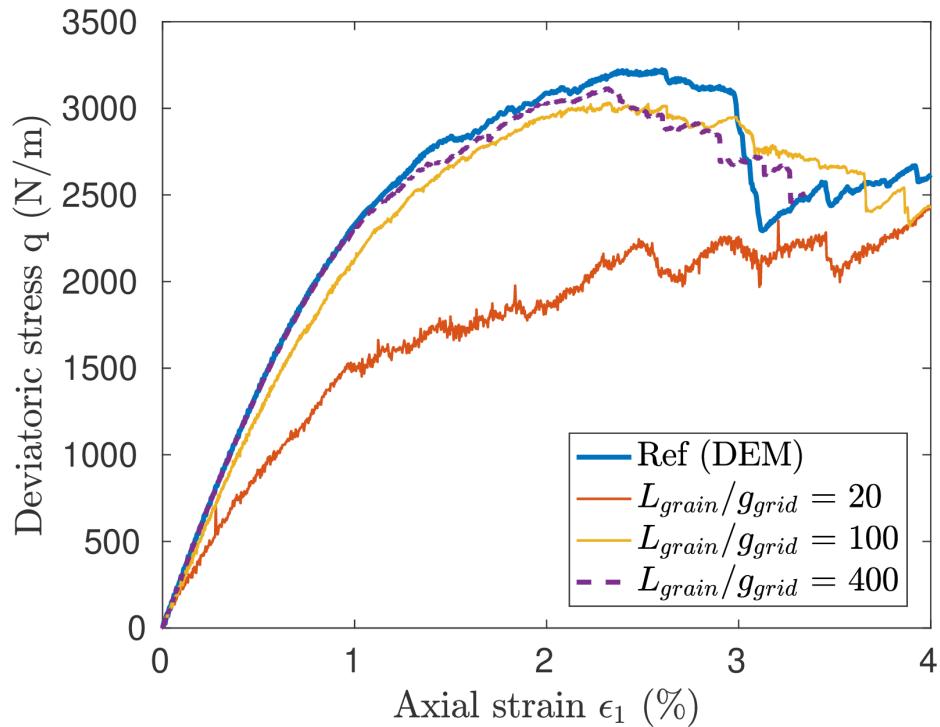
Overlap of Axis-aligned bounding boxes ?



# Validation of LS-DEM vs DEM

## Biaxial tests (1600 disks) : computational aspects

Memory cost  $\Leftrightarrow$  grid precision



# Validation of LS-DEM vs DEM

## Biaxial tests (1600 disks) : computational aspects

Time cost  $\Leftrightarrow$  number of boundary nodes

